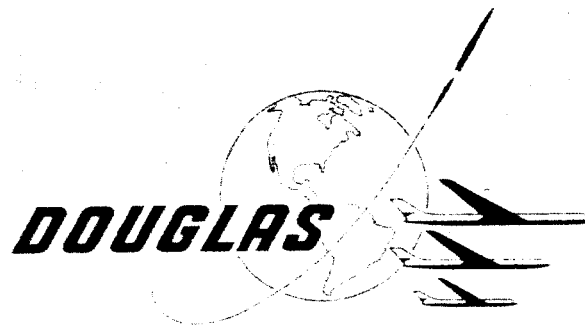# SLOSHING PARAMETERS FOR SPHERICAL TANKS

Model DSV-4

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20 July 1961

DOUGLAS

Prepared for Marshall Space Flight Center
Under Contract No. NAS7-1

Approved by: _____

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ABSTRACT

This report presents the characteristics of a mechanical system that may be used to represent the effects of fluid sloshing in spherical containers. This equivalent model may be used to simplify dynamic stability analyses which should include the effects of fluid sloshing in a spherical tank. For system stability analyses including the effects of fluid sloshing, the forces and moments produced by the sloshing fluid are of the utmost importance. It is, therefore, the forces and moments that are used as the similarity parameters between model and prototype. The model that is developed is an equivalent spring-mass system which has been used to represent the characteristics of fluid sloshing in many other types of fluid containers.

Many of the spherical tank sloshing parameters presented in this report were taken directly from Reference 1 . These parameters are indicated throughout the report.

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NOMENCLATURE

F   Sloshing force-positive right

M   Total mass of fluid in container

A   Amplitude of sinusoidal translation

λ Frequency of sinusoidal translation or rotation

ρ Mass density of fluid in container

d   Width of the free surface of the fluid

D_n Modal parameter (Reference 1)

δ_n Amplitude of nth modal slosh height at the free surface

λ_n Sloshing frequency of nth mode

g   Acceleration due to gravity

C_n Modal parameter (Reference 1)

a   Radius of spherical tank

h   Fluid height

T   Sloshing moment about origin of coordinate system-positive counterclockwise

θ Amplitude of sinusoidal rotation about origin of coordinate system

1.0 INTRODUCTION

The spherical tank sloshing parameters presented in this report were derived from equations and graphs given in Reference 1 . The results presented in this reference were analytically derived for nearly full, half full, and nearly empty spherical tanks, and interpolated for intermediate results. The natural frequencies parameters for the fundamental sloshing modes have been experimentally verified. The analytical treatment is based on an integral equation approach and makes use of the following assumptions:

1) Non viscous, incompressible fluids
2) Irrotational flow
3) Smooth wall, rigid tanks
4) Small amplitude exciting oscillations

Replacing the fluid by an equivalent spring-mass model requires consideration of the forces and moments produced on the container walls by two types of motion of the prototype and model. These motions are

1) Sinusoidal horizontal motion
2) Sinusoidal pitching about the origin of the coordinate system

For the following derivations, the origin of the coordinate system is taken as the center of the spherical tank.

2.0 SLOSHING FORCES FOR SPHERICAL TANKS

2.1 Sinusoidal Horizontal Motion

The horizontal force produced by the oscillations of fluid in a partially filled spherical tank which is under going sinusoidal translations is given by

$$1) \quad F = MA\lambda^2 \sin \lambda t - \frac{\pi \rho d^3}{8} \sum_{n=1}^{\infty} D_n \ddot{\delta}_n \qquad \text{(Reference 1)}$$

where

$$2) \quad \ddot{\delta}_n + \lambda_n^2 \delta_n = \frac{d\lambda^2 \lambda_n^2 D_n A \sin \lambda t}{2g \, C_n}$$

(Reference 1)

Combination of equations (1) and (2) yields

$$F = A\lambda^2 \sin \lambda t \left\{ M + \frac{}{16 a^2 (3-\frac{h}{a})} \sum_{n=1}^{\infty} \frac{D_n^2 \lambda_n^2}{g C_n \left[\frac{\lambda_n^2}{\lambda^2} - 1 \right]} \right\}$$

This force must pass through the center of the tank regardless of the fluid level. The net moment about the tank center is, therefore, zero.

2.2 Sinusoidal Pitching Motion

For a sinusoidal rotation of a partially filled spherical tank about the tank center, the sloshing forces and moments are zero since the fluid is assumed non-viscous.

3.0 EQUIVALENT MECHANICAL SYSTEM

3.1 Description

The mechanical system shown in Figure 1 consists of an infinite number of spring masses m_n and a fixed mass m_o. The spring masses are constrained to oscillate in the XZ plane and parallel to the X axis. Mass m_n is a point mass located at $Z = Z_n$. Mass m_o is also a point mass and is located at $Z = Z_o$.

3.2 Forces and Moments

Equations for the forces and moments produced by the mechanical system on its constraints for a sinusoidal translation of the model and for a sinusoidal rotation of the model about the origin of the coordinate system are derived in Reference 2 and repeated in the following section.

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3.3 Equivalence

The force and moment equations for fluid sloshing in a spherical container and for the mechanical system (designated by primes) are as follows:

Sinusoidal Translation:

$$F' = A\lambda^2 \sin \lambda t \left\{ M + \frac{14^4 M}{16\pi^2 (3 - \frac{h}{a})} \sum_{n=1}^{\infty} \frac{D_n^2 \lambda_n^2}{gC_n \left[\frac{\lambda_n^2}{\lambda^2} - 1 \right]} \right\}$$

$$F' = A\lambda^2 \sin \lambda t \left\{ m_0 + \sum_{n=1}^{\infty} m_n + \sum_{n=1}^{\infty} \frac{m_n}{\frac{\lambda_n^2}{\lambda^2} - 1} \right\}$$

$$T' = 0$$

$$T' = A\lambda^2 \sin \lambda t \left\{ m_0 Z_0 + \sum_{n=1}^{\infty} m_n (Z_n + g/\lambda_n^2) + \sum_{n=1}^{\infty} m_n \frac{(Z_n + g/\lambda_n^2)}{\left[\frac{\lambda_n^2}{\lambda^2} - 1 \right]} \right\}$$

Sinusoidal Pitching:

$$F' = 0$$

$$F' = \Theta\lambda^2 \sin \lambda t \left\{ m_0 Z_0 + \sum_{n=1}^{\infty} m_n (Z_n + g/\lambda_n^2) + \sum_{n=1}^{\infty} \frac{m_n (Z_n + g/\lambda_n^2)}{\frac{\lambda_n^2}{\lambda^2} - 1} \right\}$$

$$T' = 0$$

$$T' = \Theta\lambda^2 \sin \lambda t \left\{ \frac{g}{\lambda^2} \left[m_0 Z_0 + \sum_{n=1}^{\infty} m_n (Z_n + g/\lambda_n^2) \right] + m_0 Z_0^2 \right.$$

$$\left. + \sum_{n=1}^{\infty} m_n (Z_n + g/\lambda_n^2)^2 + \sum_{n=1}^{\infty} \frac{m_n (Z_n + g/\lambda_n^2)^2}{\left[\frac{\lambda_n^2}{\lambda^2} - 1 \right]} \right\}$$

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If the forces and moments produced by the mechanical system are to be equal to the forces and moments produced by fluid sloshing in the spherical tank, the characteristics of the mechanical system must be given by

$$m_n = \frac{3d^4 M D_n^2 \lambda_n^2}{16 a^2 (3-\frac{h}{a}) g C_n} = \frac{3 M a (3-\frac{h}{a})^2 D_n^2 \lambda_n^2}{g C_n (3 - \frac{h}{a})}$$

$$m_o = M - \sum_{n=1}^{\infty} m_n$$

$$Z_n = -g/\lambda_n^2$$

$$Z_o = 0$$

$$k_n = m_n \lambda_n^2 \text{ with } \lambda_n \text{ equal to the } n^{th} \text{ sloshing frequency.}$$
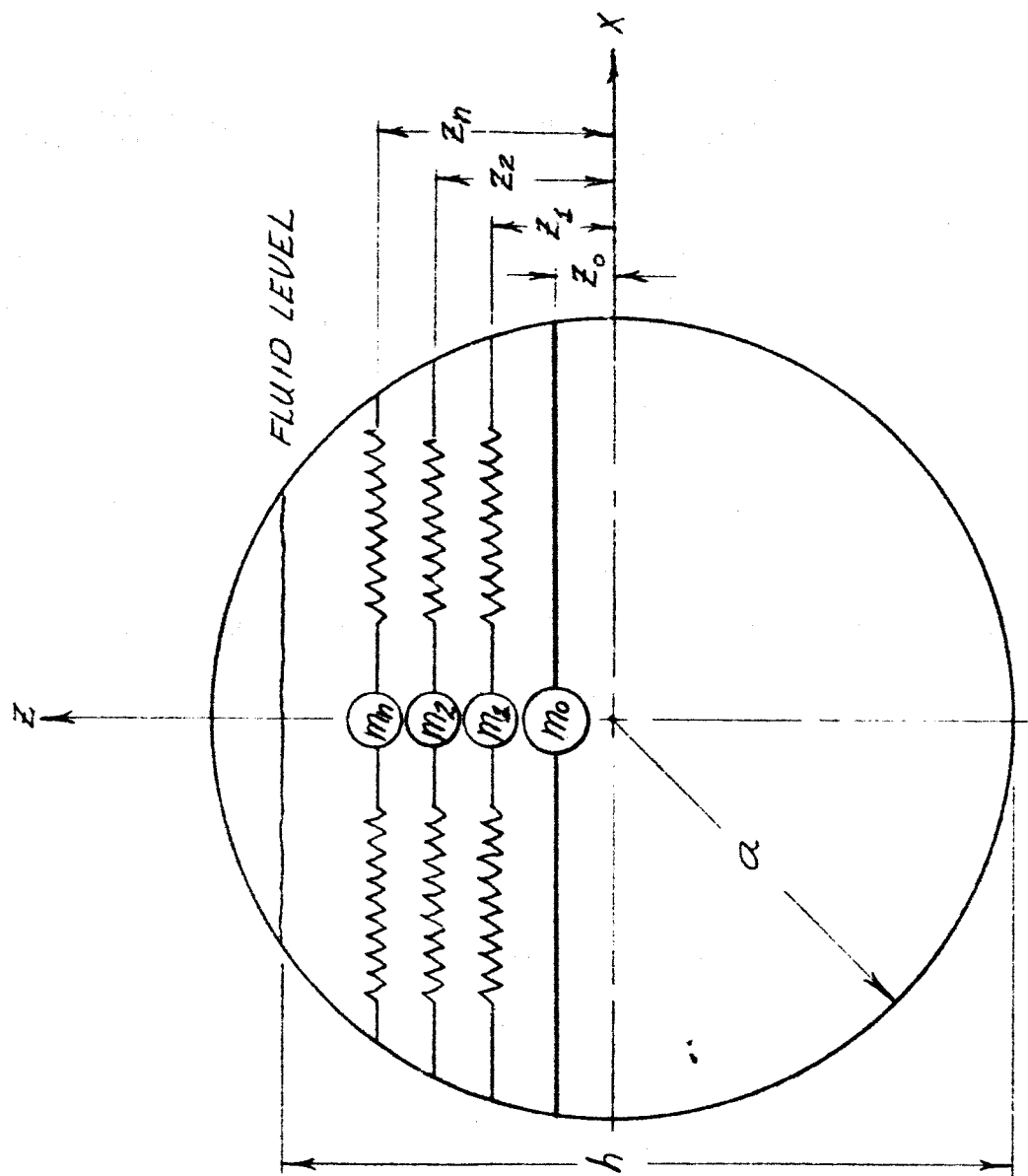
These parameters are shown graphically in Figures 1 through 4. Figure 3, Mass Characteristics of the Equivalent Mechanical System, shows that the higher modal masses are negligible when compared to the first.

4.0   CONCLUSIONS

The equivalent spring-mass model has often been used as a convenient device for studying the effects of fluid sloshing on various dynamic systems. This report has presented a suitable equivalent model for fluid contained in spherical tanks based on the work done by Bernard Budiansky. A model to represent the sloshing in circular canals could also have been presented based on the forementioned work. The derivation of this model and the results, however, are so similar that this was not given.
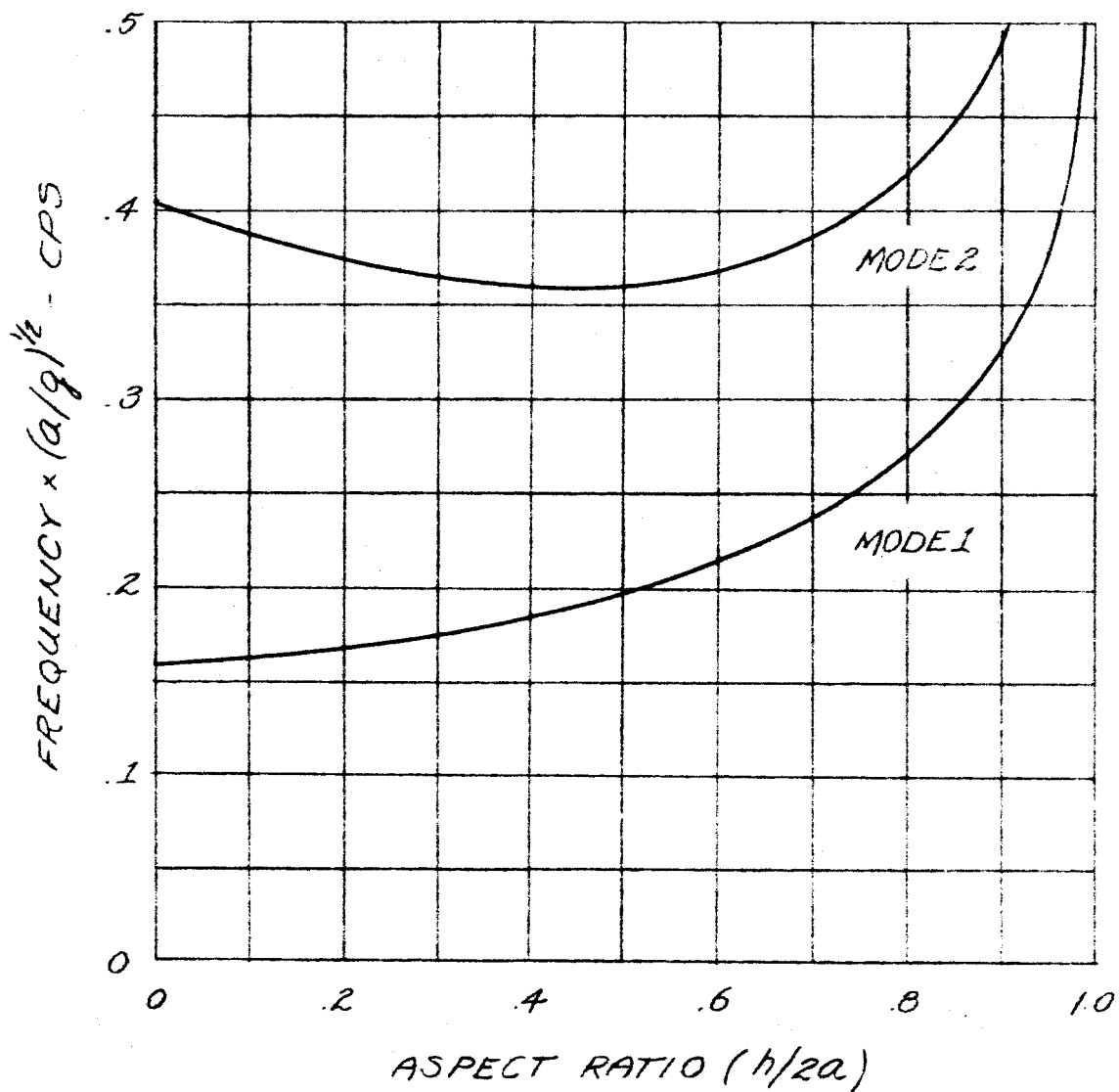
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2. Graham, E. W. and Rodriguez, A. M. "The Characteristics of Fuel Motion Which Affect Airplane Dynamics", Report No. SM-14212, Douglas Aircraft Co., Santa Monica, California, November 1951.
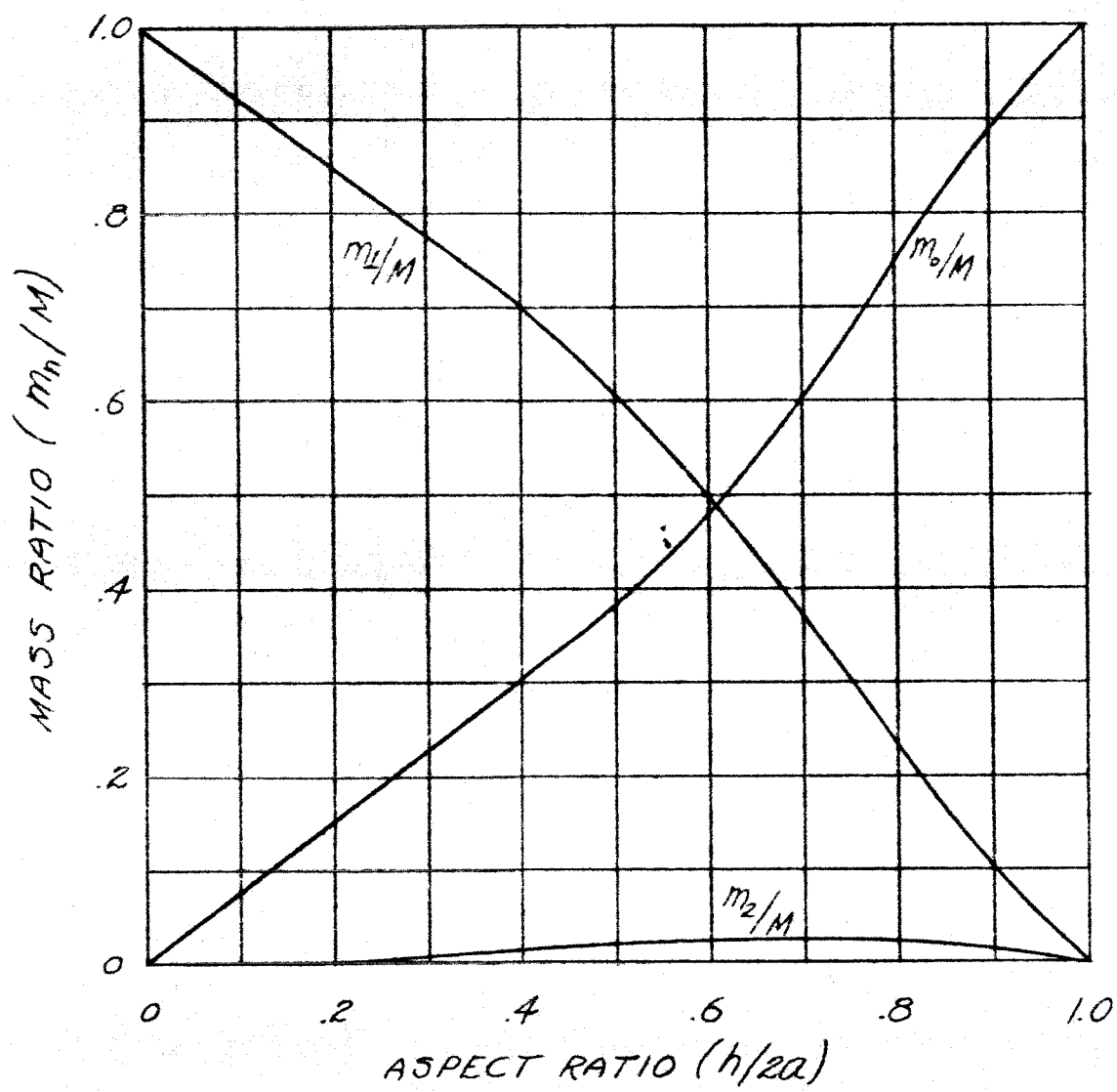
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FLUID LEVEL

EQUIVALENT MECHANICAL SYSTEM

FIGURE 1

Y-axis: FREQUENCY × $(a/g)^{1/2}$ - CPS

X-axis: ASPECT RATIO $(h/2a)$
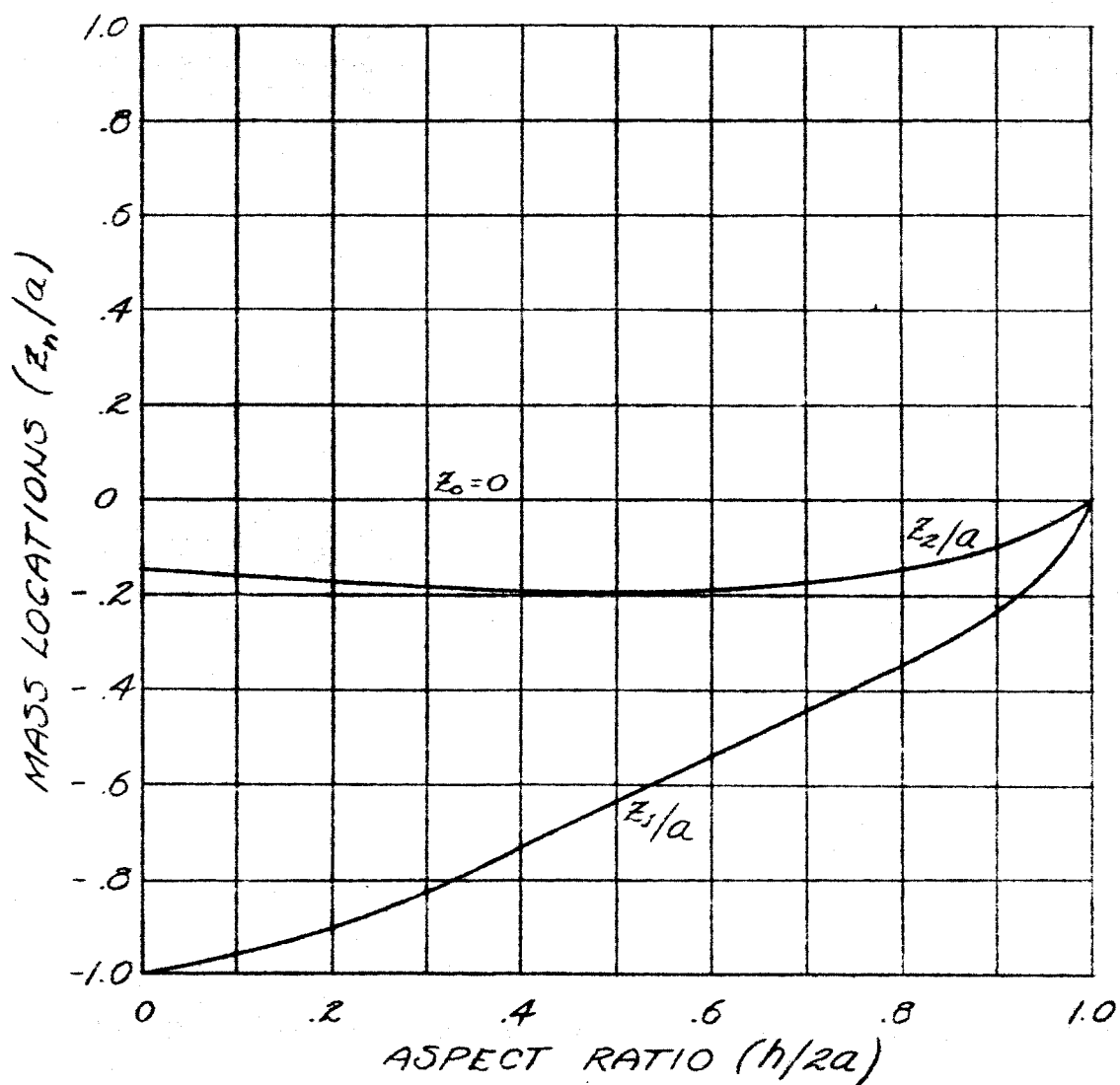
MODE 2

MODE 1

SLOSHING FREQUENCIES

FIGURE 2

MASS CHARACTERISTICS
EQUIVALENT MECHANICAL SYSTEM

FIGURE 3

MASS LOCATIONS
EQUIVALENT MECHANICAL SYSTEM

FIGURE 4